

Adiabatic Control of the Electron Phase in a Quantum Dot

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A Berry phase can be added to the wavefunction of an isolated quantum dot by adiabatically modulating a nonuniform orthogonal electric field, along a time-cycle. The dot is tuned close to a three-level degeneracy, which provides a wide range of possibilities of control. We propose to detect the accumulated phase by capacitively coupling the dot to a double-path interferometer. The effective Hamiltonian for the phase sensitive coupling is discussed in detail.

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I. INTRODUCTION

Manipulating in a controlled fashion the phase of a quantum electronic system is presently one of the most relevant challenges in nanophysics, especially in view of possible applications to quantum computing [1]. Probably, the most promising route to achieve such a task is provided by coherent solid-state devices. For instance, a superconducting Josephson qubit has already been realized as a Cooper pair box, that is, a small superconducting island, weakly coupled to a charge reservoir via a Josephson junction [2]. The quantum state of the box can be tuned to a coherent superposition of the charge-zero and the charge-one states. The possibility of realizing superpositions of flux states has been considered, as well [3]. Entanglement in a semiconducting device made out of two dots, one on top of each other (“quantum dot molecule”) has recently been optically measured [4].

Usually, quantum algorithms either assume that the system dynamically evolves through a sequence of unitary transformations, or that a set $\vec{\lambda}$ of external control parameters of the Hamiltonian H smoothly changes in time (“adiabatic evolution”) [5]. In particular, if adiabatic evolution is realized across a closed path γ in parameter space, close enough to an accidental level degeneracy, the nontrivial topology of the space makes the state of the system to take a “geometrical” phase Γ , referred to as “Berry Phase” [6]. The value of Γ may be controlled by properly choosing γ .

Following this idea, geometric adiabatic evolution has recently been proposed as a way to operate with superconducting devices without destroying phase coherence [7,8]. Another possibility is using semiconducting nanodevices, like single-electron transistors or Quantum Dots (QD). The QD state can be finely tuned, by means of external magnetic and electric fields acting on the dot, or on the coupling between the dot and the contacts [9].

Moreover, accidental level degeneracies are quite common in QD’s, as seen both theoretically, and experimentally [10]. Also, double dots have been proposed as possible qubits [11,12].

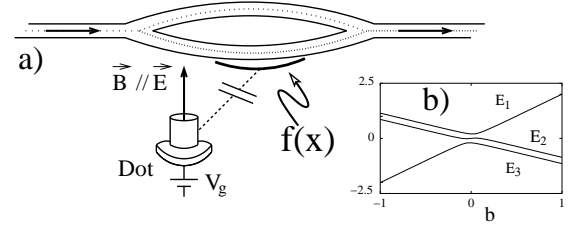


FIG. 1. **a)**: A sketch of a possible experimental setup to detect the Berry phase. $f(x)$ accounts for spatial modulation of the coupling between the dot and the interferometer (see text). **b)** Energy levels of the dot, E_i vs. $b = 3/2(B - B^*)$.

In this paper, we address in its basic points the problem of adiabatic control in a single dot, that is, *a)*: how to generate a Berry phase in an isolated dot (or in a QD at Coulomb Blockade); *b)*: how to detect it.

a) We focus onto a possible realization of the non-trivial topological structure studied in Ref. [13], but we leave aside questions concerning the details of the experimental device. After a time cycle, Γ will show up as the phase factor of the dot ground state (GS). We consider an isolated vertical QD in vertical magnetic and electric fields, B and \mathcal{E} , respectively. B and \mathcal{E} work as (adiabatic) control parameters of the system. In particular, B is tuned close to a two-level degeneracy, taking place at $B = B^*$. The adiabatic cycle is realized by keeping B fixed, and by slowly periodically varying \mathcal{E} , with time period T . Provided \mathcal{E} is nonuniform over the dot’s area, a spin-orbit coupling term arises, involving the spin of electrons at the dot. Such an interaction may give raise to a Berry phase at the QD. In particular, we will find

a nonzero Γ only if $B > B^*$, that is, only on one side of values of B , with respect to the avoided crossing point, $B = B^*$. As we will discuss in the paper, this is related to the particular form of the adiabatic dot Hamiltonian, H_D , we operate with, that is different from the usual $su(2)$ -spin Hamiltonian introduced by Berry [6].

Our model realization of the Berry phase in a quantum dot is taken as the simplest setup that can be theoretically studied, with the required features. Actually, its experimental realization is quite demanding, at least as long as one is concerned with a single dot only. For instance, one may think of a setup where the maximum of \mathcal{E} could be off-center in the dot area and rotating in time. Alternatively, slowly time-dependent asymmetries in the shape of the dot might produce a Berry phase.

Very likely, better chances of realizing adiabatic evolution, based on the same principles, may be achieved in molecular structures of strongly interacting dots [4].

b) In order to measure the phase at the dot, we propose an experimental arrangement, borrowed from an analogous experiment [14]. We imagine to capacitively couple the dot to one arm of a double-path electron interferometer (Fig.(1a)). We show how the phase carried by the transported electrons may be influenced by the dot. The dot's phase gives raise to an interference term in the total conductance across the ring.

In Appendix B, we discuss the features of the coupling between the dot and the arm of the ring, which contributes to an effective Hamiltonian for the conduction electrons in the arm coupled to the dot (hereafter referred to as “lead electrons”, as well). Such a coupling should be weak enough, not to affect the modulus of the electronic transmission amplitude. We show that the phase sensitive terms are, in general, nonlocal, both in time and in space, and that they only arise if the capacitive coupling is extended in space on a length $L \sim v_F T$, where v_F is the Fermi velocity of the electrons in the metallic lead.

In Appendix A, we derive the extra phase $\Phi(t, t')$ entering the propagator of the charge density of the dot, $G_q(t, t')$, which appears in the lead electron Hamiltonian. Because of the adiabatic cycling, time-translational invariance of $G_q(t, t')$ is lost. We compute the phase Φ , as well as Γ , and analyze in detail their expressions. A closed expression for $\Phi(t, t')$ can only be given for $t \rightarrow t' \rightarrow 0$. We find that, while Γ is determined only by the geometrical properties of H_D , Φ (given in Eq.A26), is nonlocal in time, and contains dynamical factors, that cannot be disentangled from the geometrical ones.

Finally, we argue that the essential features of the phase dependence are contained in the time dependent model Hamiltonian H_W^{eff} for lead electrons, given in Eq.(22). Using the simplified Hamiltonian H_W^{eff} allows us to straightforwardly derive the effects of the extra phase arising at the dot on the transmittance of the lead

electrons.

In conclusion, we expect that, by steadily cycling the dot Hamiltonian along a given path in parameter space, a phase difference sets in between the electrons of the two arms of the ring. The phase picked up by the electrons in moving along the arm facing the dot is averaged over a time interval L/v_F , and should be measurable as an interference contribution to the total conductance across the ring.

The very special properties of the Berry phase that we produce, allow us to discriminate whether we are measuring a phase sensitive effect or just an unwanted electrostatic influence of the dot on the conductance in the ring. Indeed, the same operation on the dot can be performed at $B > B^*$ or $B < B^*$. However, the case $B < B^*$ does not provide the lead electrons with a Berry phase. If we are really acting on the phase of the lead electrons and not on the modulus of their transmission, the interference should appear in the former case, but not in the latter.

The paper is organized as follows:

- in Section II we discuss the model Hamiltonian for the quantum dot, H_D . In particular, we specify the level structure and the control parameters of the dot. Having chosen a subspace of states appropriate to the range of parameters we are considering, we derive the corresponding eigenvalues, as functions of the tuning parameters;
- in Section III we calculate the Berry phase for the isolated dot. We show that, as the applied magnetic field B crosses B^* , there is an abrupt jump in the Berry phase from 0 to a value of order $\pi/2$. Moreover, we also show that, for $B > B^*$ the Berry phase is largely insensitive of B ;
- in Section IV we show how the Berry phase arising at the dot may be picked up by the wavefunctions of the conduction electrons in one of the arms of a ring interferometer. Using results about the adiabatic evolution in the dot, derived in Appendix A, we obtain an effective Hamiltonian for the lead electrons, including phase dependent terms (the details are reported in Appendix B);
- in Section V we discuss our results.

II. THE DOT HAMILTONIAN H_D

The dot's Hamiltonian, H_D , adiabatically depends on the external parameters B and \mathcal{E} , generically referred to as $\vec{\lambda}$, in the following. In this Section we introduce H_D and discuss how its eigenvalues and eigenstates depend on $\vec{\lambda}$.

To be specific, we consider an isolated, vertical QD, disk-shaped in the (x, y) -plane. An external static magnetic field B and an electric field \mathcal{E} are applied along

the z -axis. \mathcal{E} takes a small angular modulation: $\mathcal{E} = e + g_1 \cos(\theta) + g_2 \sin(\theta)$. (θ and ρ are the polar coordinates in the plane). The corresponding dot Hamiltonian is :

$$H_D = H_{0D} + \sum_i [\alpha^2 (\vec{\mathcal{E}} \times \vec{p}_i) \cdot \vec{\sigma}_i + \mu l_i^z B] + U(B, \omega_d) \quad (1)$$

where H_{0D} includes the confining parabolic potential of frequency ω_d , the \vec{p}_i 's and the \vec{l}_i 's are the linear and the angular momenta of the electrons at the dot, respectively. The second and third terms at the r.h.s. of Eq.(1) are the spin-orbit (Rashba) term and the Zeeman term, respectively. α^2 is the spin-orbit coupling constant and μ is the electronic magnetic moment. $U(B, \omega_d)$ is the operator for the Coulomb interaction. We neglect the Zeeman spin splitting, which is expected to be small.

The states of a vertical dot are usually denoted as $|N, M, S, S^z\rangle$, where M is the orbital angular momentum, S is the total spin and S^z is its z -component [10]. The dot state may be controlled by properly tuning the external control parameters B and \mathcal{E} .

For $N = 3$, the unpaired electron is the only one to be “active”, in a wide range of parameters. Therefore, we may label the dot's states with the quantum number of such a “valence” electron only, i.e. $|n, m, s = \frac{1}{2}, s^z\rangle$ (s, s^z are the electron spin and its z -component). Also, n, m are the orbital quantum numbers, corresponding to an orbital wavefunction for an electron in a harmonic confining potential and an external B field, given by:

$$\Psi_{n,m}(\rho, \theta) = \frac{e^{im\theta}}{l\sqrt{\pi}} R_{n|m|}(t) \quad (2)$$

with $t = \rho^2/l^2$, where $l = \sqrt{\hbar/m\omega_0}$ ($\omega_0 = \sqrt{\omega_d^2 + \omega_c^2(B)}/4$ and $\omega_c(B)$ is the cyclotron frequency). The radial wavefunction in Eq.(2) is expressed in terms of the Laguerre polynomials L_ν^μ as:

$$R_{n|m|}(t) = \mathcal{C}_{n|m|} e^{-\frac{t}{2}} t^{|m|/2} L_{\frac{n-|m|}{2}}^{|m|}(t) \quad (3)$$

$$\text{where } \mathcal{C}_{n|m|} = \left[\frac{\left(\frac{n-|m|}{2}\right)!}{\left(\frac{n+|m|}{2}\right)!} \right]^{\frac{1}{2}}.$$

Numerical diagonalization of Eq.(1) shows that, at $B = B^*$ and $\mathcal{E} = 0$, the states $|1, \vec{\lambda}\rangle \equiv |11, \frac{1}{2}, -\frac{1}{2}, \vec{\lambda}\rangle$ and $|3, \vec{\lambda}\rangle \equiv |22, \frac{1}{2}, -\frac{1}{2}, \vec{\lambda}\rangle$ become degenerate with $|2, \vec{\lambda}\rangle \equiv |11, \frac{1}{2}, \frac{1}{2}, \vec{\lambda}\rangle$, because of the $e-e$ interaction [15] (we add $\vec{\lambda}$ in the notation to stress that they depend on the value of the external parameters).

Other levels are much higher in energy, so that here we employ a 3×3 -Hamiltonian matrix, to diagonalize the spin-orbit term.

At $\mathcal{E} \neq 0$, the spin-orbit term couples states with opposite spin components: $s^z = \pm \frac{1}{2}$. The isotropic component of \mathcal{E} has matrix elements between states with

$J = M + S^z = 3/2$ (i.e. the orbital state $|3, \vec{\lambda}\rangle$, and the orbital state $|2, \vec{\lambda}\rangle$). The matrix elements of the anisotropic terms ($\propto \rho \sin \theta$ and $\propto \rho \cos \theta$) can be easily calculated. For instance, one obtains:

$$\begin{aligned} A_{n'm'+, nm-} &= \langle n'm' | \rho \sin \theta e^{-i\theta} (\partial_\rho + \frac{m}{\rho}) | nm \rangle \\ &= \frac{1}{i} (\delta_{m'+2, m} - \delta_{m', m}) \\ &\times \frac{1}{2} \int_0^\infty \sqrt{t} dt R_{n'|m'|}(t) \left(2\sqrt{t} \partial_t + \frac{m}{\sqrt{t}} \right) R_{n|m|}(t) \end{aligned} \quad (4)$$

From Eq.(4) we see that $|2, \vec{\lambda}\rangle$ is coupled to $|1, \vec{\lambda}\rangle$, with matrix element $g = g_1 - ig_2$.

Therefore, for B close to B^* , once restricted to the subspace spanned by the states $|\ell, \vec{\lambda}\rangle$ ($\ell = 1, 2, 3$), H_D can be represented by the matrix \hat{h} :

$$\hat{h}[b, g_1, g_2, e] = \begin{bmatrix} -b & 0 & g^* \\ 0 & 2b & e \\ g & e & -b \end{bmatrix}, \quad (5)$$

where $b = 3(B - B^*)/2$.

$\hat{h}[b, g_1, g_2, e]$ is a traceless 3×3 Hermitian matrix, belonging to the $su(3)$ -algebra. Its eigenvalues take a simple form in terms of the “polar” coordinates R, Ψ , defined by:

$$R = \sqrt{b^2 + \frac{e^2 + |g|^2}{3}} \quad ; \quad \sin(3\Psi) = -\frac{b(b^2 + \frac{e^2}{2} - |g|^2)}{R^3}$$

In decreasing order, the energies are given by [13]:

$$E_\ell = 2R \sin \left[\Psi + \frac{2}{3}(\ell - 1)\pi \right] \quad \ell = 1, 2, 3 \quad (6)$$

In Fig.(1b), we plot E_1, E_2 and E_3 versus b , at e, g small, but $\neq 0$. The corresponding eigenvectors will be denoted in the following by $|e_\ell, \vec{\lambda}\rangle$, $\ell = 1, 2, 3$.

Fig.(1b) shows the avoided crossing vs. b . In particular, if $b < 0$ the level E_1 is almost degenerate with E_2 , and the degeneracy at $e = g = 0$ takes place at $\Psi = \pi/6$. On the other hand, if $b > 0$, it is E_3 to be almost degenerate with E_2 , and the degeneracy takes place at $\Psi = \pi/2$. At $b = 0$ an “exceptional” three-level degeneracy arises, when $e = g = 0$ ($R = 0$), although, as discussed in detail in the next Section, this is not an accidental three-level degeneracy.

In the next Section, we show that since, for $b < 0$, there is an energy gap of order b between E_3 and the next available energy level (E_2), we do not expect any Berry phase to arise within such a region. On the other hand, we do expect a Berry phase to appear for $b > 0$, when E_3 keeps almost degenerate with E_2 . Therefore, whether a Berry phase may arise, or not, is just matter of whether b is > 0 , or < 0 . Clearly, no fine tuning of the external field is required, provided it is possible to move B across B^* .

III. CALCULATION OF THE BERRY PHASE

In this Section we calculate the Berry phase accumulated at the quantum dot, by adiabatically operating along a cycle periodic in time, with period T . We will consider a closed path γ , lying in the subspace of co-ordinates (g_1, g_2, e) , at fixed b . At $b \neq 0$, a two-level accidental degeneracy occurs when $e = g = 0$. It is possible to obtain a nonzero Berry phase if γ encircles this point.

When dealing with such a kind of problems, it is customary to define a one-parameter family of Hamiltonians, $H_A(s)$, such that $H_A(s) = H_D[\vec{\lambda}(s)]$, $\forall s$ ($s = t/T$, $s \in [0, 1]$).

The solution of the time-independent parametric Schrödinger equation associated to $H_A(s)$ provides the eigenvalues $E_l[\vec{\lambda}(s)]$ and the corresponding normalized eigenvectors $|e_l, \vec{\lambda}(s)\rangle$ (also denoted by $E_l(s)$ and $|e_l(s)\rangle$, in the following).

Let $|F(t)\rangle$ be the ground state of the system. At $t = 0$ we have $|F(0)\rangle = |e_3(0)\rangle$, that is, the ground state coincides with the ground state of the adiabatic Hamiltonian. Moreover, adiabaticity implies $|F(T)\rangle = e^{i\Gamma}|e_3(0)\rangle$, where the geometrical phase Γ is given by:

$$\Gamma = i \oint_{\gamma} d\vec{\lambda} \cdot \left\langle e_3, \vec{\lambda} \left| \vec{\nabla}_{\vec{\lambda}} \right| e_3, \vec{\lambda} \right\rangle \quad (7)$$

By means of Stoke's theorem, Γ may also be written as a two-dimensional integral on a surface \mathcal{S} , bounded by γ [6]:

$$\Gamma = \int_{\mathcal{S}} (d\lambda^a \wedge d\lambda^b) \times \sum_{\ell \neq 3} \Im m \left\{ \frac{\langle e_3, \vec{\lambda} | \frac{\partial H}{\partial \lambda^a} | e_{\ell}, \vec{\lambda} \rangle \langle e_{\ell}, \vec{\lambda} | \frac{\partial H}{\partial \lambda^b} | e_3, \vec{\lambda} \rangle}{(E_{\ell}[\vec{\lambda}] - E_3[\vec{\lambda}])^2} \right\} \quad (8)$$

where $d\lambda^a \wedge d\lambda^b$ is the projection of the surface element of \mathcal{S} on the (a, b) plane in parameter space.

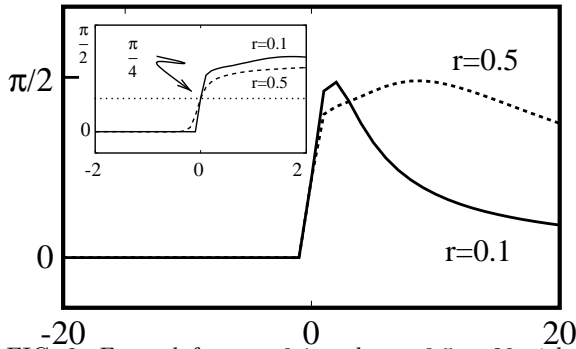


FIG. 2. Γ vs. b for $r = 0.1$ and $r = 0.5$, $-20 \leq b \leq 20$. Inset: Zoom of the plot for $-2 \leq b \leq 2$.

We have explicitly calculated Γ , in the case where γ is the closed path on the sphere of radius r defined by $g = r \sin(\vartheta)e^{i\varphi}$, $e = r \cos(\vartheta)$, $\vartheta = \pi/4$ (the detailed shape of γ is, of course, irrelevant to our calculation, and we have chosen a circular path at $\vartheta = \pi/4$ just for the sake of simplicity). We have numerically computed Γ at fixed b , $\Gamma(b)$, by using Eq.(8), where, in our case, \mathcal{S} is identified with the spherical segment $\mathcal{S} : 0 \leq \vartheta \leq \pi/4; 0 \leq \varphi \leq 2\pi$. The derivatives of \hat{h} with respect to the various parameters that matter for the calculation of Γ are provided by the following $su(3)$ -generators:

$$\frac{\partial \hat{h}}{\partial g_1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad \frac{\partial \hat{h}}{\partial g_2} = \begin{bmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{bmatrix}$$

$$\frac{\partial \hat{h}}{\partial e} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

In Fig.(2) we plot $\Gamma(b)$ vs. b . We see that, while Γ is close to zero, as long as $b < 0$, it abruptly jumps to a finite value (of order $\pi/2$) across $b = 0$. As $b > 0$, it keeps rather flat, till it starts to decrease for decreasing r/b . The extremely weak dependence of Γ on r/b for $b > 0$ is quite important for the practical purpose of detecting the Berry phase. Indeed, it is the very reason why a fine tuning of b is not required, in order to measure Γ , provided $b > 0$.

In order to explain the plot in Fig.(2) and, in particular, the behavior of $\Gamma(b)$ for $b > 0$, we have gone through an approximate analytical calculation of the Berry phase, which we are now going to discuss.

The infinitesimal contribution to the Berry phase for the system lying within the ground state of energy E_3 , $d\Gamma$, is:

$$d\Gamma \approx \Im m \sum_{\ell=1,2} \frac{1}{(E_3 - E_{\ell})^2} \times \left\{ dg_1 \wedge dg_2 \langle e_3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_1} | e_{\ell}, \vec{\lambda} \rangle \langle e_{\ell}, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_2} | e_3, \vec{\lambda} \rangle \right. \\ + dg_2 \wedge de \langle 3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_2} | e_{\ell}, \vec{\lambda} \rangle \langle e_{\ell}, \vec{\lambda} | \frac{\partial \hat{h}}{\partial e} | e_3, \vec{\lambda} \rangle \\ \left. + de \wedge dg_1 \langle e_3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial e} | e_{\ell}, \vec{\lambda} \rangle \langle e_{\ell}, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_1} | e_3, \vec{\lambda} \rangle \right\}. \quad (9)$$

Let us separately analyze the two complementary regimes $b < 0$ and $b > 0$.

- The case $b < 0$:

In this regime, up to $\mathcal{O}(r^2/b^2)$, we obtain the following approximate expressions for the energy eigenvalues:

$$\begin{aligned}
E_1 &\approx -b + |g| - \frac{e^2}{6b} \\
E_2 &\approx -b - |g| - \frac{e^2}{6b} \\
E_3 &\approx 2b + \frac{e^2}{3b}
\end{aligned} \tag{10}$$

Since $(E_3 - E_\ell)^2 \sim b^2$, it is straightforward to derive that, up to $\mathcal{O}(r^2/b^2)$, one obtains $d\Gamma_{<} = 0$ and, therefore, $\Gamma(b) = 0$, as long as $b < 0$. Such a strong suppression of the Berry phase is clearly seen from our exact numerical diagonalization results, and may be understood as a consequence of the absence, for $b < 0$, of energy levels near by E_3 .

- The case $b > 0$:

In this case, the energy eigenvalues are given by:

$$\begin{aligned}
E_1 &\approx 2b + \frac{e^2}{3b} \\
E_2 &\approx -b + |g| - \frac{e^2}{6b} \\
E_3 &\approx -b - |g| - \frac{e^2}{6b}
\end{aligned} \tag{11}$$

Since $E_2 - E_3$ is $\mathcal{O}(|g|)$, we may calculate Γ by taking into account only these two levels. The direct calculation of the various matrix elements provides us with the result:

$$\begin{aligned}
\Im m \left\{ \langle e_3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_1} | e_2, \vec{\lambda} \rangle \langle e_2, \vec{\lambda} | \frac{\partial}{\partial g_2} | e_3, \vec{\lambda} \rangle \right\} &= -\frac{e^2}{6b|g|} \\
\Im m \left\{ \langle e_3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial g_2} | e_2, \vec{\lambda} \rangle \langle e_2, \vec{\lambda} | \frac{\partial}{\partial e} | e_3, \vec{\lambda} \rangle \right\} &= -\frac{eg_1}{3b|g|} \\
\Im m \left\{ \langle e_3, \vec{\lambda} | \frac{\partial \hat{h}}{\partial e} | e_2, \vec{\lambda} \rangle \langle e_2, \vec{\lambda} | \frac{\partial}{\partial g_1} | e_3, \vec{\lambda} \rangle \right\} &= \frac{eg_2}{3b|g|}
\end{aligned} \tag{12}$$

Since $(E_3 - E_2)^2 \approx 4|g|^2$, we obtain the following expression for $d\Gamma_{>}$:

$$d\Gamma_{>} = -\frac{r}{3b} \left[1 - \frac{1}{2 \sin^2(\vartheta) + ar^2/b^2} \right] \cos(\vartheta) d\vartheta \wedge d\varphi \tag{13}$$

where a is a numerical constant that is $\mathcal{O}(1)$.

Notice that $d\Gamma_{>}$ is given by two contributions. The first one may be clearly identified with the usual $su(2)$ elementary Berry phase [6]. Indeed by introducing the polar coordinate $\vartheta' = \vartheta - \pi/2$, it takes the form:

$$d\Gamma_{>}^{(1)} = \frac{r}{3b} \sin(\vartheta') d\vartheta' \wedge d\varphi$$

where the “monopole strength” is given by $4\pi r/3b$. Once integrated, $d\Gamma_{>}^{(1)}$ provides the contribution to the finite Berry phase given by:

$$\Gamma^{(1)} = -\frac{\sqrt{2}\pi r}{3b} \tag{14}$$

that is suppressed as r/b , for large b/r .

The second contribution, on the other hand is quite peculiar of the specific problem we are dealing with, and does not take any resemblance with the usual $su(2)$ Berry phase. We have traded higher-order term appearing at the denominator for a cutoff of order of r^2/b^2 that, once integrated, provides us with the additional contribution to Γ given by:

$$\Gamma^{(2)} = \frac{\pi}{3\sqrt{a}} \text{atan} \left(\frac{\pi b}{4r} \right) \tag{15}$$

Already for $b/r \sim 5$ we may approximate $\Gamma^{(2)}$ as $\approx \pi^2/6\sqrt{a}$. The deviation of $\Gamma(b)$ from a constant value, for $b > 0$, which may be clearly seen from Fig.(2), is due to contributions that we have neglected in writing Eq.(13). Nevertheless, our result clearly shows how the peculiar form of the Berry phase in our problem makes it possible to have a finite and detectable Γ even for $r/b \sim 0.005$. Again, let us remark that this result is quite important, since it shows that no fine-tuning of B is needed, in order to give raise to a Berry phase, provided one is performing the measurement in the region $B > B_*$.

It is also possible to exactly evaluate the Berry phase at $b = 0$. This is an “exceptional” point, where $e = g = 0$ implies a three-fold degeneracy among all the energy levels, corresponding to $\Psi = \pi/3$. Such a degeneracy takes place among three states, two of which differ by the spin polarization of the unpaired electron. Hence, it is not appropriate, here, to speak of an “accidental three-level degeneracy”, since two of the three levels involved are connected by means of a continuous $SU(2)$ -symmetry. Thus, the three-level degeneracy we find in a four-parameter Hamiltonian is not inconsistent with the result of Ref. [6], where it is stated that one needs at least eight parameters, in order to get an accidental three-level degeneracy with a complex Hamiltonian [16].

Close to the three-level degeneracy, we obtain:

$$\begin{aligned}
\Gamma(b=0) &= \int_S \frac{1}{4r^4} \{ e^2 dg_1 \wedge dg_2 + eg_2 de \wedge dg_1 + eg_1 dg_2 \wedge de \} \\
&= \frac{1}{2} \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{4}} \sin(2\vartheta) d\vartheta = \frac{\pi}{4} \quad , \tag{16}
\end{aligned}$$

independently of the radius of the sphere.

In the inset of Fig.(2) we show a zoom of the region around $b = 0$. Curves for different radii intersect at $b = 0$ with a value equal to $\pi/4$, as derived in Eq.(16).

IV. DETECTION OF THE BERRY PHASE

In Section III we have calculated the total Berry phase for the isolated dot Γ . We have shown that, for the special time dependent Hamiltonian we consider, the adiabatic cycling over a period T gives $\Gamma \neq 0$ or $= 0$ depending on whether $b > 0$ or < 0 .

While one usually looks at the effects of the adiabatic cycling on the wavefunction of the isolated system (the Berry phase of the dot, in our case), in this work we want to study how the adiabatic evolution of the dot may affect the propagator for conduction electrons traveling in some conductor out of the dot, but weakly interacting with it. Therefore, our second goal of our work is to compute the relation between the electron propagator of the dot in the presence of the Berry phase cycling and the adiabatic one. This is done in Appendix A for the charge propagator:

$$G_q(t, t') = \text{Tr} [e^{-\beta H_D} \mathcal{T} U(T, t) \hat{q}(t) \hat{q}(t') U(t', T)] . \quad (17)$$

Here \mathcal{T} is the time ordering operator and $U(t, t')$ is the time-evolution operator corresponding to the dot Hamiltonian $H_D(t)$. It is given by: $U(t, t') = \mathcal{T} \exp[-i \int_{t'}^t d\tau H_D(\tau)]$. Such a calculation requires the implementation of non equilibrium Green's function approach. In Appendix A, we present such an approach. In particular, we make use of the adiabatic evolution in the QD dynamics and show that the difference between the full and the adiabatic Hamiltonian is $\mathcal{O}(1/T)$.

The phase factors containing $\Phi(t, t')$, which relate the adiabatic Green's function to the full single particle propagator are computed explicitly in Appendix A, only in the limit $t \rightarrow t'$. By comparing the derivation for Φ with the expression of Γ given by Eq.(8), we see that it is not possible to reduce the latter to the former, in any straightforward limit. Indeed, Γ is determined by geometrical factors only, while Φ contains also dynamical phase factors, which it is impossible to disentangle from the geometrical ones, due to nonlocal contributions (see e.g. Eq.(A26)). However, the derivation shows that Φ vanishes, if Γ does so. It is easily seen that the phase Φ_o of Eq.(A26) and Γ coincide only when the non-local terms are dropped.

Our third task is to find out if it is possible to detect the phase Φ by means of a conductance measurement.

In this Section, we propose an experimental way to detect a Berry phase in the dot wavefunction. We consider a double-path electron interferometer, weakly interacting with the dot by means of a capacitive coupling (see Fig.(1)). In such a kind of experimental arrangement, electrons passing through the arm of the interferometer coupled to the dot may pick up a finite phase $\Phi(t, t')$. Meanwhile, a fine tuning of the coupling should keep the modulus of the transmission of each arm roughly unitary. We show here that if the coupling between the dot and

the interferometer is dealt with perturbatively, the phase $\Phi(t, t')$ depends on the adiabatic dynamics of the isolated dot only, that is on the Green's function of Eq.(17).

In order to do so, we derive an effective Hamiltonian for lead electrons, when the interferometer arm is capacitively coupled to the dot, by integrating out the dot degrees of freedom. Since, with a point-like contact, it is impossible to effectively transfer the phase from the dot to the interferometer, we assume that the capacitive contact is extended over a length L , that is, the support of an envelope function $f(x)$ (Fig.(1a)). This gives rise to a nonlocal interaction among dot electrons. We also assume conduction electrons to propagate ballistically in the wire.

Using the results of Appendix B, we show that the nonlocal term responsible for transferring the phase from the dot to the lead can be modeled by a local effective Hamiltonian H_W^{eff} given by Eq.(22). The nonequilibrium, although adiabatic, dynamics of the system gives rise to a phase-dependent interference between wave components from different points in space, as it is evident in Appendix B, where both the time ordered and the anti-time ordered Green's functions show up in the derivation.

To deal with nonlocal interactions, we need to make use of the functional integral approach.

Let $S_D[\hat{c}(\underline{r}), \hat{c}^\dagger(\underline{r})]$ be the dot's action, derived from H_D in Eq.(1), in terms of the Fermionic fields of the dot electrons $\hat{c}(\underline{r}), \hat{c}^\dagger(\underline{r})$. Moreover, since we model the arm interacting with the dot as a one-dimensional conductor, we may expand the corresponding Fermionic field $\Psi(x)$ around the Fermi points $\pm q_F$, so that:

$$\Psi(x) = e^{-iq_F x} \psi_L(x) + e^{iq_F x} \psi_R(x) \quad (18)$$

where $\psi_{L,R}(x)$ are the left-handed (L) and the right-handed (R) component, respectively, and x is the coordinate along the wire. Since all the interactions, we are going to make use of, are diagonal in the spin index σ , we just suppress it. The free action of the wire is given by:

$$S_W = i \int dt \int dx \left[\psi_L^\dagger(x, t) \left(\frac{\partial}{\partial t} - v_F \frac{\partial}{\partial x} \right) \psi_R(x, t) + \psi_R^\dagger(x, t) \left(\frac{\partial}{\partial t} + v_F \frac{\partial}{\partial x} \right) \psi_L(x, t) \right] \quad (19)$$

where $v_F = \hbar q_F / m$ ($\hbar = m = 1$) is the Fermi velocity.

The capacitive coupling between the dot and the wire, depends on the charge density operator at the edge of the dot (parametrized by \underline{r}_b), $\hat{q}(\underline{r}_b) = \hat{c}^\dagger(\underline{r}_b) \hat{c}(\underline{r}_b)$. The coupling is assumed to smoothly extend over a distance $L \sim v_F T$ and an envelope function $f(x)$ entails the geometry of the gate facing the lead:

$$S_{\text{int}} = \int dt q(t) \int dx f(x) \Psi^\dagger(x, t) \Psi(x, t) + c.c. \quad (20)$$

(The space dependence of the charge density q is ignored because it plays no relevant role).

The effective propagator for electrons in the arm is given by:

$$G(x, t; x', t') = \int \mathcal{D}[\Psi \Psi^\dagger] \Psi^\dagger(x, t) \Psi(x', t') e^{-iS_W} \langle e^{i \int dx f(x) \int dt q(t) \Psi^\dagger(x, t) \Psi(x, t)} \rangle_{S_D}, \quad (21)$$

with appropriate boundary conditions.

To lowest order in a cumulant expansion, the average $\langle \dots \rangle_{S_D}$ over the dot variables yields:

$$\langle e^{i \int dx f(x) \int dt q(t) \Psi^\dagger(x, t) \Psi(x, t)} \rangle_{S_D} \approx e^{-\frac{1}{2} \int dt dt' G_q(t, t') \int dx dx' f(x) f(x') \Psi^\dagger(x, t) \Psi(x, t) \Psi^\dagger(x', t') \Psi(x', t')},$$

where $G_q(t, t')$ is given by Eq.(17) and discussed in Appendix A.

Eq.(21) shows that the capacitive coupling of the lead to the dot turns into an effective interaction term among the lead electrons. The corresponding action comes out to be nonlocal, both in time and in space. It is so because the problem is intrinsically nonstationary [17], although time dependence is assumed to be very slow.

We derive the effective interaction in detail in Appendix B. We first perform a mean field decoupling of the quartic term in the Fermionic fields in Eq.(21). As a next step, we approximate the kernel to be local in time, but still nonlocal in space. In this way, it takes the form of an external potential, acting on the conduction electrons and giving raise to forward and backward scattering.

As a consequence of the dot dynamics, one gets a non-local potential, whose main effects may be summarized as follows:

- a) A $L-L$ and a $R-R$ electrostatic forward scattering potential, roughly independent of the geometrical phase;
- b) A phase-dependent forward scattering term;
- c) An electrostatic $L-R$ and $R-L$ backscattering potential, roughly independent of the phase;
- d) A phase-dependent $L-R$ and $R-L$ backscattering term.

As a further step, we ignore the electrostatic perturbation induced by the dot (terms *a*) and *c*). The reason for doing so is twofold. First of all, whatever such a coupling is, its effects should not depend on whether a geometrical phase is arising at the dot ($b > 0$), or not ($b < 0$), provided the path in parameter space is the same. Moreover, we assume that the corresponding couplings are weak enough, to make the various contributions negligible anyways (of course, this requires quite accurate experimental checks, but we do not want to deal with this issue here).

On the other hand, terms *b*) and *d*) are responsible for transferring the phase from the dot to the lead. Such

terms appear in a nonlocal form, though. These are the main results of the derivation reported in Appendix B.

The full Hamiltonian describing the capacitive coupling suggests that an effective local “toy” Hamiltonian H_W^{eff} may be used, if the effects of the Berry phase are the main concern of our model construction. H_W^{eff} is given by:

$$H_W^{eff} = H_0 + V_\Phi(t) + V_\chi(t) \quad (22)$$

$$H_0 = v_F \int \frac{dk}{2\pi} k \left(\psi_L^\dagger(k) \psi_L(k) + \psi_R^\dagger(k) \psi_R(k) \right)$$

$$V_\Phi(t) = \dot{\Phi}(t) \int_{-L/2}^{L/2} dx \left(\psi_L^\dagger(x) \psi_L(x) - \psi_R^\dagger(x) \psi_R(x) \right)$$

$$V_\chi(t) = v_\chi(t) \int_{-L/2}^{L/2} dx \left(\psi_L^\dagger(x) \psi_R(x) e^{2i\Phi(t)} + h.c. \right) \quad (23)$$

Notice that H_W^{eff} in Eq.(22) explicitly depends on time. The time dependence is produced by adiabatically tuning the parameters of the dot. An expression for the coupling $v_\chi(t)$ can be derived from Eq.s (B13, B14) within the approximations used in Appendix B and involves the retarded adiabatic Green’s function, $G_{q,A}^{Ret}(t^+, t)$, defined in Appendix A. In the “local approximation” Φ depends only on t , and is independent of the space variables.

The Hamiltonian H_W^{eff} is particularly amenable because the phase Φ can be transferred onto the conduction electron wave functions in the region $(-L/2, L/2)$, by performing an unitary time dependent transformation $e^{iS(t)}$ in such space interval, with

$$S(t) = \Phi(t) \int_{-L/2}^{L/2} dx \left(\psi_L^\dagger(x) \psi_L(x) - \psi_R^\dagger(x) \psi_R(x) \right). \quad (24)$$

Indeed, it is easy to show that $e^{iS} \psi_L e^{-iS} = \psi_L e^{-i\Phi}$ and $e^{iS} \psi_R e^{-iS} = \psi_R e^{i\Phi}$, so that

$$\begin{aligned} e^{iS} \{ \partial_t - H_W^{eff} \} e^{-iS} &= \\ \partial_t - \left\{ H_0 + v_\chi(t) \int_{-L/2}^{L/2} dx \left(\psi_L^\dagger(x) \psi_R(x) + h.c. \right) \right\} & \\ \equiv \partial_t - H_{WA}. \end{aligned} \quad (25)$$

The last equality defines the adiabatic Hamiltonian H_{WA} , in which the phase Φ has disappeared (but the explicit time dependence has not!).

By acting with $e^{iS(t)}$ we relate the Green’s functions for the electrons in the lead to the adiabatic ones, $G_{A,ij}(i, j = L, R)$, corresponding to H_{WA} , by means of appropriate phase factors. In particular, if $t > t'$ and $x, x' \in (-L/2, L/2)$ we have:

$$\begin{aligned} G_{LL}(x, t; x', t') &= G_{A,LL}(x, t; x', t') e^{i \int_{t'}^t \dot{\Phi}(\tau) d\tau}, \\ G_{RR}(x, t; x', t') &= G_{A,RR}(x, t; x', t') e^{-i \int_{t'}^t \dot{\Phi}(\tau) d\tau}. \end{aligned} \quad (26)$$

Eq.s(25,26) are the third and final remarkable result of this work.

Eq.(25) defines the adiabatic Hamiltonian corresponding to our simplified model while Eq.(26) relates the electron propagators in the interferometer arm to the corresponding one of the adiabatic Hamiltonian.

Because of the phase factor appearing in the forward propagators of Eq.(26), one can infer that the total dc conductance across the ring, \mathcal{G} , picks up an extra interference contribution:

$$\mathcal{G}(\Phi) = \mathcal{G}(\Phi = 0) \times \frac{1}{2} \overline{[1 + \cos(\Phi(t+T) - \Phi(t))]} \quad . \quad (27)$$

when the dot is driven through a periodic sequence of adiabatic cycles, with time period $T \sim L/v_F$ and $b > 0$. In Eq.(27), the overline denotes time averaging over a period.

We leave a full derivation of the conductance and its numerical evaluation for a forthcoming paper. The conductance involves the transmission across opposite sides of the contact, between points at a distance $\sim L$. On the other hand the interference term is not washed out by time averaging only if the traversal time of the electrons propagating ballistically is $\sim T$. This is the reason why one should have $T \sim L/v_F$ if the interference has to be constructive.

The way in which the extra phase $\Phi(t, t')$ appears in Eq.(26) is a consequence of the local approximation made in order to relate the model Hamiltonian derived in Appendix B with the toy Hamiltonian of Eq.(22). Actually, in the computation of Appendix B, space locality had to be relaxed, in order to obtain an effective phase transfer from the dot to the wire (this requires the coupling to take place over a finite length L). Time locality, instead, is much more meaningful an approximation, since terms which are nonlocal in time may be regarded as higher time derivatives of the phase and, therefore, are negligible, due to the hypothesis of adiabaticity. This guarantees that it is always possible to write down an effective phase sensitive Hamiltonian for the capacitive coupling.

To conclude this Section, we have suggested that one has a straightforward way to detect whether or not a Berry phase arises at the dot, independently of the exact value of $\Phi(t)$, and of the detailed functional dependence of the electron propagators on Φ . Indeed, one may first tune $b < 0$, correspondingly measuring $\mathcal{G}(\Phi = 0)$. Then, moving to a working point at $b > 0$, by keeping the working condition the most possible unchanged, one may measure $\mathcal{G}(\Phi \neq 0)$ and, eventually, compare the two values of the conductance.

V. CONCLUSIONS

In conclusion, we have shown that adiabatically operating on three levels of an isolated quantum dot provides

quite a wider range of possibilities than operating on two levels only. In particular, a nonzero Berry phase Γ appears, or not, according to whether $B > B^*$, or $B < B^*$.

It has already been noticed that spin-orbit interaction may generate a Berry phase [18]. Indeed, electrons moving along a ring in an applied electric field \mathcal{E} feel, in their reference frame, an effective magnetic field, which gives raise to spin precession. In our case, the static B field applied on the dot provides a cyclotron motion, by changing the direction of the effective magnetic field along the electronic trajectories.

Our first important result is that there is no obvious relation between the Berry phase arising at the isolated dot, Γ , and the “dynamical” phase Φ , appearing in the electron propagator. This is quite a crucial point of our analysis. Indeed, while Γ is the geometrical phase acquired by the total wavefunction of dot electrons, Φ is not a purely geometrical effect, since it includes dynamical features, that cannot be disentangled from the geometrical ones. As shown in Appendix A, this is a consequence of nonlocality of quantum electron propagation. As a matter of fact, the phase Φ vanishes, if Γ does so.

In Section IV we describe a way of detecting the extra phase arising at the dot by means of a double path interferometer. The setting we propose in Fig.(1a) is quite peculiar, in that it works as a Bohm-Aharonov interferometer, although with no oscillating magnetic flux threading the ring. Instead, our interferometer is controlled by voltage tuning at the quantum dot, which is located outside of the ring.

In Appendix B, we show that an effective phase transfer mechanism needs a capacitive coupling extended in space. A local approximation on the model Hamiltonian derived in Appendix B leads us to the toy Hamiltonian H_W^{eff} (Eq.(22)), which allows for a simple description of the transferral of the phase to conduction electrons. Despite its apparent simplicity, the potential in Eq.(23) is able to capture the relevant physics arising when there is a nonzero Berry phase at the dot, and to derive the consequences for the dc conduction properties of the ring in quite a straightforward way. In particular, because of quantum interference, phase sensitive terms in the electron propagator do not disappear, as one takes the local limit.

From the derivation in Appendix B, it is clear that, in order to transfer the adiabatic phase from the dot electrons to the lead electrons, the following inequality has to be fulfilled

$$\frac{\hbar}{E_2 - E_3} < T \sim \frac{L}{v_F} \quad .$$

The former inequality is the adiabaticity condition: $E_2 - E_3$ is the separation between the dot levels, that is $\mathcal{O}(|g|)$, for $B > B^*$. It means that cycling must not cause crossings between dot levels. On the other hand,

the propagator for lead electrons takes essential contributions nonlocal in space, due to electron self-interference. In order for such an interference to carry informations about the total Berry phase at the dot, the electron wavefunction must interfere with its opposite chirality component, “coming” from a distance $L \sim v_F T$.

A point of view alternative to ours is found in Ref. [19], where the conduction electrons are thought of as an environment for the dot, in the presence of a Berry phase. The bath degrees of freedom are integrated out, to provide the adiabatic variable with a stochastic “force”. In our scheme, instead, we keep the environment degrees of freedom (conduction electrons) and calculate the effects of the Berry phase on their dynamics.

A device different from ours has been considered in Ref. [20], where a QD is embedded in one arm of the interferometer. Usually, such a setup does not break phase coherence [14]. In Ref. [20], however, a quantum point contact (QPC) is facing the dot, to detect single-charge tunneling across the dot as a change in its transmission. The detection provides a sudden perturbation and an orthogonality catastrophe in the dot arm, which affects Aharonov-Bohm interference in the ring. This is exactly the limit opposite to ours, since we manipulate the phase of the conducting electrons by adiabatically operating on a quantum dot at fixed charge. Moreover, while the dot-QPC coupling of Ref. [20] is point-like, in our case, it is extended over a length L .

Finally it is important to remark that we have excluded any pumping effect in the wire [21].

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APPENDIX A: ADIABATIC EVOLUTION OF THE GREEN’S FUNCTIONS OF THE ISOLATED DOT.

In Section IV we propose to detect the Berry phase of the dot by electrostatically weakly coupling the dot to a conducting wire located near by. In Appendix B we will derive the effective Hamiltonian for conduction electrons arising from such a coupling, after integrating out dot’s coordinates. In particular, the additional couplings involve the Green’s functions for the charge density operator of the isolated dot, $\hat{q}(\underline{r}_b)$, like, for instance, the time-ordered Green’s function:

$$G_q(t, \underline{r}_b; t', \underline{r}'_b) = -i \text{Tr} \left[\hat{\rho} \mathcal{T} \hat{q}(t, \underline{r}_b) \hat{q}(t', \underline{r}'_b) \right] \quad (\text{A1})$$

where $\hat{\rho}$ is the density matrix for the isolated dot and \mathcal{T} is the time ordering operator.

In this Appendix we will derive the Green’s functions for the isolated dot undergoing adiabatic evolution of Eq.(A1). In particular, we will show how G_q is related to its adiabatic counterpart, $G_{q,A}$, constructed from the adiabatic Hamiltonian H_A .

Since the coordinate \underline{r}_b , confined to the boundary of the dot, is immaterial in the following, we just drop it henceforth.

The adiabatic Hamiltonian $H_A(s)$ has been introduced in Section III. Its eigenstates, $|e_m(s)\rangle$, solve, with eigenvalues $E_m(s)$, the stationary Schrödinger equation for $H_D(t)$ (shortly denoted by $H(t)$ in the following) with t fixed and equal to Ts :

$$H_A(s)|e_m(s)\rangle = E_m(s)|e_m(s)\rangle. \quad (\text{A2})$$

In Section III we have shown that, when the isolated dot is driven along a closed path in parameter space, $\gamma = \vec{\lambda}(s)$ ($0 \leq s \leq 1$), its state $|F(t=0)\rangle$ evolves into $|F(T)\rangle$, by picking a Berry phase Γ , that is, $|F(T)\rangle = e^{i\Gamma}|F(0)\rangle$. Γ is given by (see Eq.(7)):

$$\Gamma = i \oint_{\gamma} d\lambda^a \left\langle e_0, \vec{\lambda} \left| \frac{\partial}{\partial \lambda^a} \right| e_0, \vec{\lambda} \right\rangle. \quad (\text{A3})$$

In Eq.(A3), $|e_0, \vec{\lambda}\rangle$ coincides with $|e_3, \vec{\lambda}\rangle$ of Section III. The sum over repeated indices a is understood.

In order to relate the full Hamiltonian $H(t)$ to H_A , let us consider the projector $P_m(s) = |e_m(s)\rangle\langle e_m(s)|$. Since during the adiabatic evolution, level crossings are forbidden, we have $[H(Ts), P_m(s)] = 0$. P_m evolves in time according to:

$$P_m(s) = U_A^\dagger(s) P_m(0) U_A(s) \quad \forall s \quad (\text{A4})$$

where $U_A(s) = U_A(s, 0) = \mathcal{T} \exp[-iT \int_0^s d\sigma H_A(\sigma)]$.

By following the derivation of Ref. [22], it is possible to prove that $H_A(s)$ is related to $H(sT)$ according to:

$$H_A(s) = H(Ts) + \frac{i}{T} \sum_m [\dot{P}_m(s), P_m(s)] \quad (\text{A5})$$

where $\dot{P}_m(s) = \frac{d}{ds} P_m(s)$.

In order to compute the time derivatives of $P_m(s)$, let us employ the integral representation of the projector in terms of the analytic extension of the Green’s operator to complex energies, $G_A(z; s) = (z - H_A(s))^{-1}$:

$$P_m(s) = \oint_{r_m} \frac{dz}{2\pi i} G_A(z; s). \quad (\text{A6})$$

where r_m is a closed path encircling $E_m(s)$.

From Eq.(A6), it is straightforward to prove that:

$$\dot{P}_m(s) = \oint_{r_m} \frac{dz}{2\pi i} G_A(z; s) \dot{\lambda}^a(s) \frac{\partial H}{\partial \lambda^a} G_A(z; s), \quad (\text{A7})$$

Therefore, we obtain the following representation for $[\dot{P}_m, P_m]$:

$$[\dot{P}_m, P_m](s) = \sum_{n(n \neq m)} \left\{ |e_n(s)\rangle \frac{\langle e_n(s) | \dot{\lambda}^a \frac{\partial H}{\partial \lambda^a} | e_m(s) \rangle}{\omega_{mn}(s)} \langle e_m(s) | \right. \\ \left. - |e_m(s)\rangle \frac{\langle e_m(s) | \dot{\lambda}^a \frac{\partial H}{\partial \lambda^a} | e_n(s) \rangle}{\omega_{mn}(s)} \langle e_n(s) | \right\} \quad (\text{A8})$$

where $\omega_{mn}(s) = E_m(s) - E_n(s)$. In the following part of this Appendix, we will use the short-hand notation $|m_s\rangle$, rather than $|e_m(s)\rangle$.

Eq.(A8) shows, in particular, that $[\dot{P}_m, P_m]$ is a fully off-diagonal operator, as it must be. In the following, we will use its off-diagonal matrix elements:

$$\langle m | [\dot{P}(s), P(s)] | n \rangle = -2 \frac{\dot{\lambda}_s^a}{\omega_{mn}} \langle m | \frac{\partial H}{\partial \lambda^a} | n \rangle \quad (m \neq n). \quad (\text{A9})$$

Another important operator is $\Omega(s) = U_A^\dagger(s) U(Ts)$. It may be shown that $\Omega(s) = 1 + \mathcal{O}(1/T)$, but its derivative is $\mathcal{O}(1)$:

$$\dot{\Omega}(s) = -U_A^\dagger(s) \sum_m [\dot{P}_m(s), P_m(s)] U_A(s) \Omega(s) \quad (\text{A10})$$

The time-ordered Green's function for the observable \hat{q} may be written as:

$$G_q(t, t') = -i\theta(t - t') \text{Tr} \left[\hat{\rho} \Omega^\dagger(s) \hat{q}_A(s) \Omega(s) \Omega^\dagger(s') \hat{q}_A(s') \Omega(s') \right] \\ - i\theta(t' - t) \text{Tr} \left[\hat{\rho} \Omega^\dagger(s') \hat{q}_A(s') \Omega(s') \Omega^\dagger(s) \hat{q}_A(s) \Omega(s) \right]$$

where we have written down the time ordering prescription explicitly. The adiabatic operator is defined as $\hat{q}_A(s) = U_A(s) \hat{q} U_A^\dagger(s)$.

To $\mathcal{O}(\frac{1}{T})$, the equation of motion for $G_q(t, t')$ will contain terms depending on $\dot{\Omega}(s)$:

$$i\partial_t G_q(t, t') = \text{Tr} \left[\mathcal{T} [H_A(s), \hat{q}_A(s)] \hat{q}_A(s') \rho_\Omega(s') \right] \\ + \delta(t - t') + \text{Tr} \left[\mathcal{T} \rho_\Omega(s') \Omega(s') \dot{\Omega}^\dagger(s) q_A(s) \Omega(s) \Omega^\dagger(s') q_A(s') \right] \\ + \text{Tr} \left[\mathcal{T} \rho_\Omega(s') \Omega(s') \Omega^\dagger(s) q_A(s) \dot{\Omega}(s) \Omega^\dagger(s') q_A(s') \right]$$

The statistical weight is $\rho_\Omega(s') = \Omega(s') \rho \Omega^\dagger(s')$. To lowest order in $1/T$ it coincides with the equilibrium density matrix ρ at $t = 0$.

Finite-temperature Wick theorem allows us to write down the second and third line at the r.h.s. of Eq.(A11) as:

$$i\text{Tr} \left[\mathcal{T} \Omega(s') \dot{\Omega}^\dagger(s) \rho_\Omega(s') \right] G_{q,A}(s, s') \\ + i\text{Tr} \left[\mathcal{T} \dot{\Omega}(s) \Omega^\dagger(s') \rho_\Omega(s') \right] \tilde{G}_{q,A}(s, s') \quad , \quad (\text{A12})$$

where we have introduced the time-ordered Green's function of $\hat{q}_A(s)$, $G_{q,A}$, and the anti time-ordered Green's

function, $\tilde{G}_{q,A}$. This amounts to evaluate the average along a Keldysh contour [23]. $G_{q,A}$ and $\tilde{G}_{q,A}$ are explicitly given by:

$$G_{q,A}(s, s') = -i\text{Tr} \left[\mathcal{T} \hat{\rho}(T) \hat{q}_A(s) \hat{q}_A(s') \right] \\ \tilde{G}_{q,A}(s, s') = -i\text{Tr} \left[\tilde{\mathcal{T}} \hat{\rho}(0) \hat{q}_A(s) \hat{q}_A(s') \right] \quad (\text{A13})$$

where $\tilde{\mathcal{T}}$ is the anti-time ordering operator.

In order to close the set of equations of motion, we need the analog of Eq.(A11) for \tilde{G}_q :

$$i\partial_t \tilde{G}_q(t, t') = \text{Tr} \left[\tilde{\mathcal{T}} \hat{\rho}(0) [H_A(s), \hat{q}_A(s)] \hat{q}_A(s') \right] \\ - \delta(t - t') + i\text{Tr} \left[\tilde{\mathcal{T}} \dot{\tilde{\Omega}}(s) \tilde{\Omega}^\dagger(s') \rho_{\tilde{\Omega}}(s') \right] \tilde{G}_{q,A}(s, s') \\ + i\text{Tr} \left[\tilde{\mathcal{T}} \rho_{\tilde{\Omega}}(s') \tilde{\Omega}(s') \dot{\tilde{\Omega}}^\dagger(s) \right] G_{q,A}(s, s')$$

where $\tilde{\Omega}(s) = U_A(s, 0) U(0, s)$ and $\rho_{\tilde{\Omega}}(s') = \tilde{\Omega}(s') \rho \tilde{\Omega}^\dagger(s')$. Let us define :

$$\partial_s \Phi(s, s') \equiv i\text{Tr} \left[\mathcal{T} \dot{\Omega}(s) \Omega^\dagger(s') \rho_\Omega(s') \right] \quad (\text{A14})$$

It is immediately seen that

$$\partial_s \tilde{\Phi}(s, s') \equiv i\text{Tr} \left[\tilde{\mathcal{T}} \dot{\tilde{\Omega}}(s) \tilde{\Omega}^\dagger(s') \rho_{\tilde{\Omega}}(s') \right] = \partial_s \Phi^\dagger(s, s') \quad (\text{A15})$$

With the definitions of Eq.s (A14,A15), the set of equations of motion for the G_q 's now reads:

$$i\partial_t G(t, t') = \delta(t - t') + H_A(s) G_A(s, s') \\ + \left[\partial_s \tilde{\Phi}(s, s') G_A(s, s') - \partial_s \Phi(s, s') \tilde{G}_A(s, s') \right] ; \\ i\partial_t \tilde{G}(t, t') = -\delta(t - t') + H_A(s) \tilde{G}_A(s, s') \\ + \left[\partial_s \Phi(s, s') G_A(s, s') - \partial_s \tilde{\Phi}(s, s') G_A(s, s') \right] \quad (\text{A16})$$

We now use the definitions:

$$G^>(t, t') - \tilde{G}(t, t') \equiv G^{(R)}(t, t') \\ G^<(t, t') - \tilde{G}(t, t') \equiv G^{(A)}(t, t') \\ (\text{A11}) \quad \tilde{G}(t, t') + \tilde{G}(t, t') \equiv G^{(K)}(t, t') \equiv G^>(t, t') + G^<(t, t').$$

Subtracting, for $s > s'$, the two Eq.s(A16), we obtain:

$$i\partial_t G^{(R)}(t, t') = \delta(t - t') + H_A(s) G_A^{(R)}(s, s') \\ + \left(\partial_s \tilde{\Phi}(s, s') - \partial_s \Phi(s, s') \right) G_A^{(K)}(s, s') \quad (\text{A17})$$

We proceed in the same way for the advanced Green's function $G^{(A)}$, in the case $s < s'$.

The sum of Eq.s (A16) gives:

$$i\partial_t G^{(K)}(t, t') = H_A(s) G_A^{(K)}(s, s') \\ + \left(\partial_s \Phi(s, s') + \partial_s \tilde{\Phi}(s, s') \right) G_A^{R,A}(s, s') \quad (\text{A18})$$

where the retarded or advanced Green's function appears on the r.h.s. depending on whether $s > s'$, or $s < s'$.

The derivatives w.r.to t' can be worked out in the same way. By changing t and t' , Ω^\dagger and Ω are exchanged. This implies a minus sign:

$$i\partial_{t'}G^{(R)}(t, t') = \delta(t - t') + G_A^{(R)}(s, s')H_A(s') - \left[\partial_{s'}\tilde{\Phi}(s, s') - \partial_s\Phi(s, s') \right] G_A^{(K)}(s, s') \quad (\text{A19})$$

Let us define:

$$\partial_s\varphi_\pm(s, s') \equiv \partial_s\tilde{\Phi}(s, s') \pm \partial_s\Phi(s, s') . \quad (\text{A20})$$

From now on we choose $s > s'$. Using the cyclic invariance of the trace we see that:

$$\begin{aligned} \partial_s\Phi(s, s') &= -i \left\langle F_{0s'} \left| U_A(s', s) [\dot{P}(s), P(s)] \right| F_{s0} \right\rangle \\ \partial_s\tilde{\Phi}(s, s') &= i \left\langle F_{1s'} \left| U_A(s', s) [\dot{P}(s), P(s)] \right| F_{s1} \right\rangle , \end{aligned}$$

where $|F_{s0}\rangle = U(s, 0)|o\rangle$ ($|F_{s1}\rangle = U(s, 1)|o\rangle$), and we assume we are working at zero temperature.

We now calculate $|F_{s0}\rangle$, $|F_{s1}\rangle$.

They solve the Schrödinger equation:

$$i \frac{d|F_s\rangle}{ds} = TH|F_s\rangle \quad (\text{A21})$$

with different initial conditions. We expand the ground state on the adiabatic basis set:

$$\begin{aligned} |F_{s1}\rangle &= U(s, 1)|F_1\rangle = e^{i\Gamma} \left\{ e^{-iT \int_1^s E_o} |o_s\rangle \right. \\ &\quad \left. + \sum_{l \neq o} a_{lo}(s) e^{-iT \int_1^s E_l} |l_s\rangle \right\} \\ |F_{s0}\rangle &= U(s, 0)|F_0\rangle = \left\{ e^{-iT \int_0^s E_o} |o_s\rangle \right. \\ &\quad \left. + \sum_{l \neq o} b_{lo}(s) e^{-iT \int_0^s E_l} |l_s\rangle \right\} \end{aligned}$$

where Γ is the Berry phase. By inserting these states in Eq.(A21), we obtain the differential equations for the coefficients a_{lo} , b_{lo} . These are solved perturbatively, by taking, to zero order, $a_{lo}^0(s) = b_{lo}^0(s) = \delta_{lo}$. Using the initial condition $a_{lo}(1) = \delta_{lo}$, we find:

$$a_{lo}(s) = \int_s^1 \dot{\lambda}_{s'}^a ds' \frac{e^{iT \int_{s'}^1 \omega_{lo}}}{\omega_{lo}} \langle l_{s'} | \frac{\partial H}{\partial \lambda^a} | o_{s'} \rangle \quad l \neq o \quad (\text{A22})$$

in the first case. In the same way, with $b_{lo}(0) = \delta_{lo}$, we get:

$$b_{lo}(s) = - \int_0^s \dot{\lambda}_{s'}^a ds' \frac{e^{iT \int_0^{s'} \omega_{lo}}}{\omega_{lo}} \langle l_{s'} | \frac{\partial H}{\partial \lambda^a} | o_{s'} \rangle \quad l \neq o \quad (\text{A23})$$

in the second case.

By inserting Eq.s (A9,A22,A23) in $\varphi_\pm(s, s')$, given by Eq.(A20), to lowest order we obtain the following four terms:

$$\begin{aligned} \partial_s\varphi_+ &= -i \left(\int_{s'}^1 + \int_0^{s'} \right) Z_s^* + i \left(\int_s^1 + \int_0^s \right) Z_s \\ &\equiv -2 \Im m \int_0^1 d\tau Z_s(\tau) \equiv \dot{\Phi}_o \\ \partial_s\varphi_- &= -i \left(\int_{s'}^1 - \int_0^{s'} \right) Z_s^* + i \left(\int_s^1 - \int_0^s \right) Z_s \end{aligned} \quad (\text{A24})$$

with

$$\begin{aligned} Z_s(\tau) &= 2 \dot{\lambda}_s^a \dot{\lambda}_\tau^b \sum_{m \neq o} \langle o_\tau | \frac{\partial H}{\partial \lambda^b} | m_\tau \rangle \\ &\quad \times \frac{e^{-iT \int_s^\tau d\tau' \omega_{mo}}}{(\omega_{mo})^2} \langle m_s | \frac{\partial H}{\partial \lambda^a} | o_s \rangle \end{aligned} \quad (\text{A25})$$

$\dot{\Phi}_o$ is independent of s, s' because of our assumption that $\Omega(s)\Omega^\dagger(s') \approx 1$.

The integral of $\dot{\Phi}_o$ w.r.to s can be rewritten as:

$$\Phi_o = 2 \Im m \int_0^1 ds \int_0^1 ds'' (\dot{\lambda}^a(s) \dot{\lambda}^b(s'')) \sum_{m \neq 0} \quad (\text{A26})$$

$$\langle e_0(s'') | \frac{\partial H}{\partial \lambda^a} | e_m(s'') \rangle \frac{e^{iT \int_s^{s''} d\tau \omega_{0m}(\tau)}}{(\omega_{mo})^2} \langle e_m(s) | \frac{\partial H}{\partial \lambda^b} | e_0(s) \rangle ,$$

where the symbols are defined in Eq.(8). We see that Φ_o resembles the result of Eq.(8), but it takes the form of a non local Berry phase.

On the other hand $\partial_s\varphi_- \rightarrow \dot{\Phi}_o$ only in the limit $s' \rightarrow 0^-$, $s \rightarrow 0^+$:

$$\partial_s\varphi_-(s \rightarrow 0^+, s' \rightarrow 0^-) \rightarrow \dot{\Phi}_o \quad (\text{A27})$$

In the limit $s' \rightarrow 0^-$, $s \rightarrow 0^+$, the motion equations become:

$$\begin{aligned} \mathcal{L}(G^> - G^<) &= \dot{\Phi}_o (G^> + G^<) \\ \mathcal{L}(G^> + G^<) &= \dot{\Phi}_o (G^> - G^<) \end{aligned} \quad (\text{A28})$$

where we have used the short hand notation \mathcal{L} , to denote the differential operator of Eq.s (A16). This implies that:

$$\mathcal{L}G^> = \dot{\Phi}_o G^> , \quad \mathcal{L}G^< = -\dot{\Phi}_o G^< ,$$

or

$$G^{(>,<)}(t, t') = G_A^{(>,<)}(s, s') e^{\pm i(t-t') \cdot \dot{\Phi}_o} \quad (s \sim s' \sim 0, s > s') \quad (\text{A29})$$

Hence, $\dot{\Phi}_o$ appears in the phase factor relating the dot's Green's functions to the adiabatic ones, at least at small time intervals. When $s - s'$ increases, $G^{(>)}$ and $G^{(<)}$ are mixed together and the solution of the system of Eq.(A17, A18) is not straightforward.

APPENDIX B: THE EFFECTIVE HAMILTONIAN FOR THE ELECTRONS IN THE LEAD

In this Appendix we calculate the effective propagator for electrons in the arm facing the dot, given by Eq.(21). By integrating out the dot degrees of freedom, we construct an effective action. This action describes an interaction among lead electrons, mediated by the quantum dot. Here, we will show that it may be approximated by a mean field self-consistent effective potential, acting on the lead electrons, and we derive in detail the various contributions. This shows that the model Hamiltonian in Eq.(22) does, in fact, entail the relevant features of the Φ -dependent term in the effective Hamiltonian.

As we have outlined in Section IV, we perform the integration over dot's variables within Feynman's action formalism. The following four-Fermion effective action arises:

$$S_{int} = -\frac{1}{2} \int dt dt' G_q(t, t') \int dx dx' \times f(x) f(x') [\Psi^\dagger(x, t) \Psi(x, t) \Psi^\dagger(x', t') \Psi(x', t')] . \quad (B1)$$

$G_q(t, t')$ appearing in Eq.(B1) is the time ordered Green's function for the isolated dot, that we have derived in Appendix A.

Since we are approximating the wire as a one-dimensional conductor, we may write the Fermionic field as a linear combination of the left-handed and of the right-handed fields, introduced in Eq.(18):

$$\Psi(x) = e^{-iq_F x} \psi_L(x) + e^{iq_F x} \psi_R(x) \quad (B2)$$

We decouple the four-Fermion term in Eq.(B1) by means of a Hubbard-Stratonovitch transformation. Then, we perform a saddle-point approximation, where we express the auxiliary Bosonic fields in terms of self-consistent pairwise averages of the Fermionic fields. In particular, in order to make the derivation the simplest is possible, we take the Green's functions $\langle\langle \psi_i(x, t) \psi_j^\dagger(x', t') \rangle\rangle$, ($i, j = L, R$) to be equal to the ones for electrons scattered by a point-like scattering center at the origin.

The various kernel appearing in the effective action are nonlocal, both in time and space. In order to resort to an Hamiltonian that is local in time, we single out the most relevant trajectories of conduction electrons, by taking the local-time limit ("local-time approximation"). Following such a procedure gives raise to an effective $L-L$ and $R-R$ "forward" scattering contribution, as well as to $L-R$ and $R-L$ "backscattering" terms. We expand for $x \sim x'$ close to the origin where the overlap $f(x)f(x')$ is maximum. In the local approximation we assume that Eq. (A29) can be extensively used. Let us now show the details of the derivation:

term a) : A first contribution to the $L-L$ scattering is given by the following decoupling pattern:

$$S_{LL}^{(a)} = -\frac{1}{2} \int dt dt' \int dx dx' f(x) f(x') G_q(t, t') \times \left\{ \psi_L^\dagger(x, t) \psi_L(x', t') [\langle\langle \psi_L(x, t) \psi_L^\dagger(x', t') \rangle\rangle + \langle\langle \psi_R(x, t) \psi_R^\dagger(x', t') \rangle\rangle e^{2iq_F(x-x')}] - \psi_L^\dagger(x', t') \psi_L(x, t) [\langle\langle \psi_L^\dagger(x, t) \psi_L(x', t') \rangle\rangle + \langle\langle \psi_R^\dagger(x, t) \psi_R(x', t') \rangle\rangle e^{-2iq_F(x-x')}] \right\} . \quad (B3)$$

Here we take $\langle\langle \psi_{L(R)}(x, t) \psi_{L(R)}^\dagger(x', t') \rangle\rangle = -i/[x - x' \mp v_F(t - t') + i0^+]^{-1}$ (where the upper (lower) sign is for $L(R)$), and $\langle\langle \psi_{L(R)}^\dagger(x, t) \psi_{L(R)}(x', t') \rangle\rangle = i[x - x' \mp v_F(t - t') + i0^+]^{-1}$. We estimate $G_q(t, t')$ in the local-time approximation, at $|t - t'| \approx |x - x'|/v_F$. Only the principal part \mathcal{P} of the integral survives in Eq.(B3), giving:

$$S_{LL}^{(a)} = \int dt \int dx dx' f(x) f(x') \psi_L^\dagger(x, t) \psi_L(x', t) \times e^{iq_F(x-x')} \mathcal{P} \left[\frac{\sin q_F(x-x')}{x-x'} \right] \times \left[G_q(t + \frac{x-x'}{v_F}, t) + G_q(t - \frac{x-x'}{v_F}, t) \right] . \quad (B4)$$

We further expand the term in Eq.(B4) involving the dot Green's functions about $x - x' \sim 0$, to give the final result:

$$S_{LL}^{(a)} = 2 \int dt \int dx dx' f(x) f(x') \psi_L^\dagger(x, t) \psi_L(x', t) \times e^{iq_F(x-x')} \mathcal{P} \left[\frac{\sin(q_F(x-x'))}{x-x'} \right] \Re\{G_{q,A}^{\text{Ret}}(t^+, t)\} \quad (B5)$$

A similar term arises if the same decoupling pattern is used for the right-handed field. Putting both together, we obtain an explicitly time-dependent Hamiltonian contribution in the form:

$$V_a(t) = v_a(t) \int dx dx' f(x) f(x') \mathcal{P} \left[\frac{\sin q_F(x-x')}{x-x'} \right] e^{iq_F(x-x')} [\psi_L^\dagger(x, t) \psi_L(x', t) + \psi_R^\dagger(x, t) \psi_R(x', t)] \quad (B6)$$

where $v_a(t) = \Re\{G_{q,A}^{\text{Ret}}(t^+, t)\}$.

The contribution above is a purely electrostatic forward scattering potential, independent of Φ . As it is not affected by the adiabatic phase, we have dropped it in the discussion of the effective Hamiltonian of Section IV, Eq.(22).

term b) : A second possible decoupling pattern contributes to an $L-L$ scattering potential, as well:

$$S_{LL}^{(b)} = -\frac{1}{2} \int dt dt' \int dx dx' f(x) f(x') G_q(t, t') \times \left\{ \psi_L^\dagger(x, t) \psi_L(x', t') [\langle\langle \psi_R(x, t) \psi_L^\dagger(x', t') \rangle\rangle e^{2iq_F x} + \langle\langle \psi_L(x, t) \psi_R^\dagger(x', t') \rangle\rangle e^{-2iq_F x'}] \right\}$$

$$-\psi_L^\dagger(x', t') \psi_L(x, t) [\langle \langle \psi_R^\dagger(x, t) \psi_L(x', t') \rangle \rangle e^{-2iq_F x} + \langle \langle \psi_R^\dagger(x, t) \psi_L(x', t') \rangle \rangle e^{2iq_F x'}] \quad (\text{B7})$$

Here we take $\langle \langle \psi_{L(R)}(x, t) \psi_{R(L)}^\dagger(x', t') \rangle \rangle = i/[x + x' \mp v_F(t - t') - i0^+]^{-1}$. In the local-time approximation, we consider only fluctuations around $|x + x'| \sim \pm v_F |t - t'|$. Following the same steps as before we get:

$$S_{LL}^{(b)} = \int dt \int dx dx' f(x) f(x') \psi_L^\dagger(x, t) \psi_L(x', t) e^{iq_F(x-x')} \mathcal{P} \left[\frac{\sin(q_F(x+x'))}{x+x'} \right] \times \left[G_q(t + \frac{x+x'}{v_F}, t) - G_q(t - \frac{x+x'}{v_F}, t) \right] \quad (\text{B8})$$

Differently from Eq.(B4), Eq.(B8) contains the difference between the G_q 's. This gives raise to a first-order contribution in $\dot{\Phi}$, when expanding the last row of Eq.(B8), according to Eq.s(A29). The result is:

$$S_{LL}^{(b)} = \frac{2}{v_F} \int dt \Im m \{ G_{q,A}^{\text{Ret}}(t^+, t) \} \dot{\Phi}(t) \times \int dx dx' f(x) f(x') \sin(q_F(x+x')) \psi_L^\dagger(x, t) \psi_L(x', t) \quad (\text{B9})$$

where $2 \Im m \{ G_{q,A}^{\text{Ret}}(t^+, t) \} / v_F$ is a slowly-varying function of time.

Since the right-moving fields enter the forward scattering term with the same expression, except for the replacement $v_F \rightarrow -v_F$, the total Φ -dependent forward scattering potential for the lead electrons will be:

$$\tilde{V}_\Phi(t) = \dot{\Phi}(t) \int dx dx' f(x) f(x') \frac{2}{v_F} \Im m \{ G_{q,A}^{\text{Ret}}(t^+, t) \} \sin(q_F(x+x')) [\psi_L^\dagger(x, t) \psi_L(x', t) - \psi_R^\dagger(x, t) \psi_R(x', t)]$$

This term is phase sensitive and corresponds to the local potential $V_\Phi(t)$ in H_W^{eff} (Eq.(23)).

term c):

Here, we consider the effective action generated by the contraction pattern:

$$S_{LR}^{(c)} = -\frac{1}{2} \int dt dt' G_q(t, t') \int dx dx' f(x) f(x') \times \{ \psi_L^\dagger(x, t) \psi_R(x', t') [\langle \langle \psi_L(x', t') \psi_L^\dagger(x, t) \rangle \rangle e^{2iq_F x'} + \langle \langle \psi_R(x', t') \psi_R^\dagger(x, t) \rangle \rangle e^{2iq_F x}] - \psi_L^\dagger(x', t') \psi_R(x, t) [\langle \langle \psi_L^\dagger(x, t) \psi_L(x', t') \rangle \rangle e^{2iq_F x} + \langle \langle \psi_R^\dagger(x, t) \psi_R(x', t') \rangle \rangle e^{2iq_F x'}] \} \quad (\text{B10})$$

In the local-time approximation, we get:

$$S_{LR}^{(c)} = \int dt \int dx dx' f(x) f(x') e^{iq_F(x+x')} \times \psi_L^\dagger(x, t) \psi_R(x', t) \mathcal{P} \left[\frac{\sin(q_F(x-x'))}{x-x'} \right] \times \left[G_q(t + \frac{x-x'}{v_F}, t) - G_q(t - \frac{x-x'}{v_F}, t) \right] \quad (\text{B11})$$

Expansion of these term gives zero, when $x \rightarrow x'$. term d):

The last contraction pattern gives raise to the following term:

$$S_{LR}^{(d)} = -\frac{1}{2} \int dt dt' G_q(t, t') \int dx dx' f(x) f(x') \{ \psi_L^\dagger(x, t) \psi_R(x', t') [\langle \langle \psi_L(x, t) \psi_R^\dagger(x', t') \rangle \rangle + \langle \langle \psi_R(x, t) \psi_L^\dagger(x', t') \rangle \rangle e^{2iq_F(x+x')}] - \psi_L^\dagger(x', t') \psi_R(x, t) [\langle \langle \psi_R^\dagger(x, t) \psi_L(x', t') \rangle \rangle + \langle \langle \psi_L^\dagger(x, t) \psi_R(x', t') \rangle \rangle e^{2iq_F(x+x')}] \} \quad (\text{B12})$$

In the local-time approximation, Eq.(B12) takes the approximate form:

$$S_{LR}^{(d)} = \int dt \int dx dx' f(x) f(x') \psi_L^\dagger(x, t) \psi_R(x', t) e^{iq_F(x+x')} \mathcal{P} \left[\frac{\sin q_F(x+x')}{x+x'} \right] \times \left[G_q(t + \frac{x+x'}{v_F}, t) + G_q(t, t - \frac{x+x'}{v_F}) \right] \quad (\text{B13})$$

Here the arguments of the two G_q 's to be added at the last line of Eq.(B13) are swapped with respect to those appearing in Eq.(B4). Hence, the sum now reads:

$$G_q(t + \frac{x+x'}{v_F}, t) + G_q(t, t - \frac{x+x'}{v_F}) \quad (\text{B14})$$

$$\approx 2 \exp \left[\frac{i}{2} \int_{t-(x+x')/v_F}^{t+(x+x')/v_F} d\tau \dot{\Phi}(\tau) \right] G_{q,A}^{\text{Ret}}(t^+, t)$$

This shows that the term of Eq.(B13) corresponds to the local potential $V_\chi(t)$ in H_W^{eff} (Eq.(23)).

The complex conjugate of Eq.(B13) is obtained, when working out the contraction patterns giving raise to the $R - L$ -backscattering term.

In conclusion, in constructing the effective potential arising from capacitive coupling between arm and dot, we have identified two different forward scattering terms, a) and b), and two backscattering contributions, c) and d). Terms a) and c) contribute to an electrostatic potential which is roughly Φ -independent and are disregarded in Section IV. In Section IV, we just retain terms b) and d), and trade them for the local Hamiltonian H_W^{eff} of Eq.(22). Despite its apparent simplicity, H_W^{eff} is able to catch the relevant asymptotic behavior of our physical system: Φ -dependent phase shifts and corresponding phase factors in the reflection and transmission amplitudes. This is the key motivation for using the simple local potential of Eq.(23), rather than the involute nonlocal scattering potential arising from the analysis reported here.

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